

# Dark Energy from graviton-mediated interactions in the QCD vacuum

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## Abstract

Adopting the hypothesis about the exact cancellation of vacuum condensates contributions to the ground state energy in particle physics to the leading order in graviton-mediated interactions, we argue that the observable cosmological constant can be dynamically induced by an uncompensated quantum gravity correction to them. To start with, we demonstrate a possible cancellation of the quark-gluon condensate contribution to the total vacuum energy density of the Universe without taking into account the graviton-mediated effects. In order to incorporate the latter, we then calculate the leading-order quantum correction to the classical Einstein equations due to metric fluctuations induced by the non-perturbative vacuum fluctuations of the gluon and quark fields in the quasiclassical approximation. It has been demonstrated that such a correction to the vacuum energy density has a form  $\varepsilon_\Lambda \sim G\Lambda_{\text{QCD}}^6$ , where  $G$  is the gravitational constant, and  $\Lambda_{\text{QCD}}$  is the QCD scale parameter. We analyze capabilities of this approach based on the synthesis between quantum gravity in quasiclassical approximation and theory of non-perturbative QCD vacuum for quantitative explanation of the observed Dark Energy density.

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## I. INTRODUCTION

The existence of mysterious Dark Energy which drives current accelerated expansion of the Universe is confirmed in many cosmological observations so far, e.g. in studies of the type Ia Supernovae [1], cosmic microwave background anisotropies [2], large scale structure [3] etc. The hypothesis about the time-independent cosmological constant called  $\Lambda$ -term is an essential part of the Standard Cosmological Model known as the Cold Dark Matter with  $\Lambda$ -term Model (or  $\Lambda$ CDM) and agrees well with all observational data collected so far at the current level of experimental accuracy. However, theoretical origin of the cosmological constant has not been properly understood yet.

Nowadays, the Dark Energy problem remains one of the major unsolved problem of theoretical physics [4]. On the way of searching for possible solutions of this problem many various pathways were explored during last few decades referring to e.g. new exotic forms of matter (e.g. “quintessence” [5], “phantom” [6] etc), holographic models [7], string theory landscape [8, 9], Born-Infeld quantum condensate [10], modified gravity approaches [11, 12] etc. For a comprehensive overview of existing theoretical models and interpretations of the Dark Energy, see e.g. Refs. [13–16] and references therein. In this work we are primarily focused on the class of those approaches which are based upon conventional quantum field theory and standard quantum gravity in quasiclassical approximation.

The traditional identification of the  $\Lambda$ -term satisfying the equation of state  $P_\Lambda = -\varepsilon_\Lambda$  with vacuum pressure  $P_\Lambda$  and energy density  $\varepsilon_\Lambda$  suffers from a large gap of knowledge on non-perturbative dynamics of the ground state in particle physics. In particular, individual vacuum condensates e.g. those which are responsible for the chiral (quark-gluon condensate) and gauge (Higgs condensate) symmetries breaking in the Standard Model contribute to the vacuum energy of the Universe individually exceeding the observable value of the Dark Energy by many orders of magnitude in absolute value [17]. This situation (sometimes referred to as the “Vacuum Catastrophe” in the literature) requires extra hypotheses about (partial or complete) compensation of vacuum condensates of different types to the net vacuum energy density of the Universe (see e.g. Ref. [9]). Note that most of the models developed in the literature adopt the same point of view trying to cancel or suppress short distance vacuum fluctuations in one way or another (for a review on such models, see e.g. Ref. [13] and references therein). Still, a dynamical mechanism for such gross cancellations and corresponding major fine-tuning of vacuum parameters is not known at the moment and is a subject of intensive studies in the vast literature (see e.g. Ref. [18]).

Historically, about 45 years ago Ya. B. Zeldovich has pointed out in Ref. [19] that cosmological constraints on the value of the  $\Lambda$ -term density to a good accuracy correspond to a simple purely mass dimensional estimate  $\varepsilon_\Lambda \sim Gm^6$  (in natural units  $\hbar = c = 1$ ), where  $G = M_{Pl}^{-2}$  is the gravitational constant and  $m$  is some characteristic mass of light elementary particles (or light hadrons known at that time). For the first time, he proposed an interpretation of the  $\Lambda$ -term as an effect of gravitational interactions of virtual particles in the physical vacuum. Almost at the same time, A. D. Sakharov noticed in Ref. [20] that, indeed, extra terms describing an effect of graviton exchanges between identical particles (e.g. bosons in the ground state) should appear in the right hand side of Einstein equations averaged over their quantum ensemble. Below we refer to this approach as the Zeldovich-Sakharov (ZS) scenario.

One of the well-known and naive representations of the Zeldovich relation  $\varepsilon_\Lambda \sim Gm^6$  through the basic fundamental constants, the minimal (typical hadron scale) and maximal

(Planck scale) scales of fundamental particle physics, has been proposed by N. S. Kardashev in Ref. [21]. It has the following form

$$\varepsilon_\Lambda = \frac{m_\pi^6}{(2\pi)^4 M_{Pl}^2} \simeq 3.0 \times 10^{-35} \text{ MeV}^4, \quad (1.1)$$

where  $m_\pi \simeq 138 \text{ MeV}$  is the pion mass [22],  $M_{Pl} = G^{-1/2} \simeq 1.22 \cdot 10^{22} \text{ MeV}$  is the Planck mass. It is worth to notice that the representation (1.1) turns out to be numerically very close to the most recent WMAP data, well within experimental error bars,  $\varepsilon_\Lambda^{\text{exp}} = (3.0 \pm 0.7) \times 10^{-35} \text{ MeV}^4$  [2]. Such a remarkable numerical coincidence of the naive Zeldovich-Kardashev (ZK) formula (1.1) to the current cosmological observations seems to be almost too good to be just an accident [23]. This situation yet remains puzzling and requires a closer look into possible physical reasons for that. A deeper theoretical understanding of those reasons is the major goal of our current work.

It is well-known that the pion mass is an object of essentially non-perturbative Quantum Chromodynamics (QCD) driven by properties of the quark-gluon vacuum condensate [24], the strongest non-perturbative vacuum sub-system at the minimal energy scales known in particle physics. So the numerical coincidence of the ZK result (1.1) to the observational data if not accidental poses a question about a possible role of strong non-perturbative vacuum fluctuations of quark and gluon fields in the  $\Lambda$ -term generation in the framework of ZS scenario described above [19, 20].

Among existing quantum field theory approaches describing the observable  $\Lambda$ -term density value, the most promising and successful ones are recently proposed by Klinkhamer and Volovik in Ref. [27] and by Urban and Zhitnitsky in Ref. [28]. These approaches attempt to interpret the positive Dark Energy density as a small but non-vanishing effect of gravitating non-perturbative QCD vacuum fluctuations in a non-trivial background of expanding Universe. The first approach [27] is based upon a generic “ $q$ -theory” operating with a conserved microscopic  $q$  value, whose statics and dynamics are studied at macroscopic scales and which can, in principle, be identified with the gluon condensate in QCD. In this case it has been shown explicitly that the gravitating non-perturbative QCD vacuum fluctuations dynamically generate a nonzero limiting value of the vacuum energy density in the nonequilibrium context of the expanding Universe. The second approach [28] focuses on dynamics of the ghost fields in the low energy (chiral) QCD. In particular, it was shown that the Veneziano ghost, being unphysical in the usual Minkowski QFT, exhibits non-vanishing physical effect in the expanding universe leading to a positive vacuum energy density with a time-dependent equation of state. Both approaches arrive at similar order-of-magnitude estimates for the  $\Lambda$ -term density in the expanding Universe in terms of the maximal  $M_{Pl}$  and minimal  $\Lambda_{QCD}$  scales of particle physics whose existence is required by the unitarity condition of an underlined quantum theory [28]. Similar QCD-based ideas of the Dark Energy origin from the effective interacting gluon condensate were previously explored in Refs. [25]. Also, quantum effects in the Born-Infeld fields condensation very similar to the gluon condensation in QCD (realized by means of the trace anomaly) which could potentially play an important role in the dynamical  $\Lambda$ -term generation were discussed in Ref. [10]. Another interesting claim was made in Ref. [26], where it was demonstrated that an extra contribution to the vacuum energy density of the form  $\sim G\Lambda_{QCD}^6$  may also originate from a consideration of QCD in spacetime with torsion in the framework of the Einstein-Cartan theory with minimal fermion coupling to torsion. Such promising QCD-based interpretations of the Dark Energy give us a strong motivation for further investigations in this direction and requires development of

a rigorous self-consistent theoretical framework based upon quasiclassical (semiquantum) gravity and theory of non-perturbative QCD vacuum fluctuations, in their existing standard formulation.

The basic ideas of this paper can be condensed into a few sentences as follows. In the zeroth order in metric fluctuations, the net energy density of non-perturbative QCD vacuum fluctuations to the energy density of the Universe can be, in principle, *compensated* via a *macroscopic* spatially-homogeneous modes of the cosmological Yang-Mills fields discussed by us recently in Ref. [29] or a *microscopic* dynamical mechanism of the QCD vacuum self-tuning. The latter possibility will be considered below in Section II<sup>1</sup>. Further, in Section III and IV we demonstrate that the non-perturbative vacuum fluctuations of quark and gluon fields *dynamically* induce the fluctuations of the background metric in the expanding Universe through a coupling of gravity to the gluon field via the *trace anomaly*. Such a coupling leads to an extra correction term in the energy-momentum tensor which is *linear* in the graviton field (as the first-order quantum gravity correction in the quasiclassical approximation). The existing (static) instanton theory of the non-perturbative QCD vacuum [31] (for a review on the QCD theory of instantons see e.g. Ref. [32]), however, does not enable us to constrain the precise numerical value of the QCD contribution to the  $\Lambda$ -term density fully dynamically, based on the First Principles only. Nevertheless, it is possible, in fact, to find some specific conditions and theoretical constraints under which the QCD vacuum theory leads to the observed  $\Lambda$ -term density value. Two of these conditions inherent to QCD, the presence of the conformal anomaly and a strong coupling theory, have already been emphasized e.g. in Refs. [23, 33]. Besides, as will be shown in Section V and VI, under certain phenomenological constraints known from the chiral QCD theory and lattice simulations the extra quantum gravity correction term indeed gives rise to an additional positive non-vanishing  $\Lambda$ -term-type contribution to the vacuum energy density of the Universe close to the observable value, in accordance with the Sakharov hypothesis [20].

## II. QCD VACUUM ENERGY: ZEROth ORDER IN METRIC PERTURBATIONS

The ground state in QCD is typically characterized by non-vanishing condensates of strongly interacting quarks and gluons commonly referred to as *the quark-gluon condensate* responsible for the confined phase of quark matter. In the framework of the instanton liquid models, the *topological (or instanton) modes* of the quark-gluon condensate are given essentially by the strong non-perturbative fluctuations of the gluon and sea (mostly, light) quark fields which are induced in processes of quantum tunneling of the gluon vacuum between topologically different classical states [32]. The topological instanton-type contribution  $\varepsilon_{vac(top)}$  to the energy density of the QCD vacuum is one of its main characteristics [24] and can be derived from the well-known trace anomaly relation [34]

$$T_{i(QCD)}^i = \frac{\beta(g_s^2)}{2} F_{ik}^a F_a^{ik} + \sum_{q=u,d,s} m_q \bar{q}q, \quad (2.1)$$

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<sup>1</sup> Possible mechanisms for compensation of other *perturbative* contributions to the net vacuum energy density of the Universe from virtual fermions and bosons (e.g. Higgs condensates, graviton condensate etc) typically refer to Supersymmetry and high-scale Grand Unified Theories [13, 30] and will not be discussed further in this paper.

where  $T_{i(\text{QCD})}^i$  is the trace of the energy-momentum tensor,  $\alpha_s = g_s^2/4\pi$ ,  $m_q$  are the light current sea quark masses, and  $F_{ik}^a$  is the gluon field strength tensor in the standard normalisation. A vacuum average of the trace  $\langle 0|T_{i(\text{QCD})}^i|0\rangle = 4\varepsilon_{\text{vac}(\text{top})}$  finally leads to the well-known formula for the instanton energy density (see e.g. Refs. [35, 36])

$$\begin{aligned} \varepsilon_{\text{vac}(\text{top})} = & -\frac{9}{32}\langle 0| : \frac{\alpha_s}{\pi} F_{ik}^a(x) F_a^{ik}(x) : |0\rangle + \frac{1}{4} \left[ \langle 0| : m_u \bar{u}u : |0\rangle + \langle 0| : m_d \bar{d}d : |0\rangle \right. \\ & \left. + \langle 0| : m_s \bar{s}s : |0\rangle \right] \simeq -(5 \pm 1) \times 10^9 \text{ MeV}^4, \end{aligned} \quad (2.2)$$

composed of gluon and light sea  $u, d, s$  quark contributions. This is the saturated (maximal) value of the topological contribution to the QCD vacuum energy density, while possible refinements to it are dependent on poorly known non-perturbative long-range Yang-Mills dynamics and were discussed e.g. in Ref. [36].

Clearly, contributions of a different physical nature should compensate the QCD contribution (2.2) to the vacuum energy of the Universe since its value by far is not compatible with the cosmological observations and data on the  $\Lambda$ -term density value [2]. On the other hand, within the general problem of vacuum condensates cancellation and corresponding fine tuning of vacuum substructures, the contribution (2.2) has a special status. Various existing cancellation mechanisms refer essentially to an unknown high-scale physics beyond the Standard Model e.g. to Supersymmetry [13, 30]. However, they cannot be applied for a compensation of the specifically non-perturbative and low-energy QCD contribution given by Eq. (2.2). This issue forces us to look for alternative cancellation mechanisms.

One of the possible ways to eliminate the *microscopic* QCD vacuum contribution (2.2) to the vacuum energy density of the Universe was discussed earlier by us in Ref. [29] introducing the hypothesis about the existence of the cosmological *macroscopic* Yang-Mills fields in early Universe. In particular, it was claimed that the negative QCD contribution (2.2) can, in principle, be canceled by a positive constant contribution generated by spatially-homogeneous modes of such a field. Corresponding quasiclassical solution for these modes is exact, it necessarily takes into account interactions with the QCD vacuum (vacuum polarisation effects) and gives rise to the spatially-homogeneous finite-time instantons.

In this paper, we consider another approach to the compensation of the topological QCD contribution assuming that there might be extra contributions from the *spatially-inhomogeneous* modes of a different nature to the QCD vacuum density at the QCD energy scale  $\sim \Lambda_{\text{QCD}}$  besides the topological (instanton) ones given by Eq. (2.2) and spatially-homogeneous ones predicted in Ref. [29]. In fact, the importance of such extra *long-range quantum-wave* fluctuations of the *hadronic vacuum* has been emphasized earlier, e.g. in Ref. [37]. Within the alternative *microscopic* approach to compensation of the QCD contribution, the total QCD vacuum energy density may turn into zero due to a fine-tuning of the QCD vacuum parameters corresponding to quantum-topological and quantum-wave fluctuations in a vicinity of the  $\Lambda_{\text{QCD}}$  scale. Let us explore such a possibility in detail.

The quantum-topological fluctuations contributing to the QCD vacuum energy density (2.2) exist at typical space-time scales  $\sim l_g$  which satisfy the following approximate inequality [32]

$$\begin{aligned} l_{g(\text{min})} & \lesssim l_g < l_{g(\text{max})}, \\ l_{g(\text{min})} & \simeq (1500 \text{ MeV})^{-1}, \quad l_{g(\text{max})} \simeq (500 \text{ MeV})^{-1}. \end{aligned} \quad (2.3)$$

Here, the values of  $l_{g(\text{min})}$ ,  $l_{g(\text{max})}$ , which can be estimated e.g. in the lattice QCD framework, are interpreted as the minimal and maximal length scales of the non-perturbative gluon field

fluctuations and approximately correspond to boundaries of the light resonances region in the hadron spectrum [24]<sup>2</sup>.

In general, besides topological (instanton-type) fluctuations contributing to Eq. (2.2), there are other two types of quantum-wave QCD vacuum fluctuations: (1) *perturbative* fluctuations of gluon and quark fields at smaller length scales  $l < l_{g(min)}$ , and (2) vacuum fluctuations corresponding to *collective wave motion* of gluon and quark fields at the same scales as the instanton ones (2.3) [37]. As was mentioned earlier, the problem of compensation of the short-range perturbative QCD fluctuations, along with all other high energy vacuum subsystems corresponding to e.g. zero-point fluctuations of fundamental fields and Higgs-type condensates, is the subject of a supersymmetric “Theory of Everything” and therefore is not discussed here. Meanwhile, quantum-wave fluctuations of the second type (from now on, we denote them as the *collective* ones) have quantum numbers of hadrons with masses  $m_h \leq l_{g(min)}^{-1}$ . Their *renormalized* Lorentz-invariant contribution to the net QCD vacuum energy density is then expressed in terms of the light hadron masses and the universal cut-off parameter  $\mu \simeq l_{g(min)}^{-1}$  (playing a role of the renormalisation scale) as follows [24]

$$\varepsilon_{vac(h)} = \frac{1}{32\pi^2} \left( 2 \sum_B (2J_B + 1) m_B^4 \ln \frac{\mu}{m_B} - \sum_M (2J_M + 1) m_M^4 \ln \frac{\mu}{m_M} \right). \quad (2.4)$$

where  $J_B$  and  $J_M$  ( $m_B$  and  $m_M$ ) are the spins (masses) of respective baryon and meson degrees of freedom, respectively.

Note, both topological and collective fluctuations have a non-perturbative nature. Strictly speaking, the expression (2.2) based upon the trace anomaly relation (2.1) should contain both the quantum-topological and collective contributions. Their separate consideration, however, is necessary since they have a completely different structure [37] while there is no a self-consistent theory of the non-perturbative QCD vacuum dynamics which could enable us to extract the collective-wave contribution (2.4) from the general expression (2.2). To this end, we formally remove the quantum-wave fluctuations from Eq. (2.2) taking the saturated (maximal) value for the topological contribution as was formally denoted by the symbols of normal ordering.

The most important observation here is that taking into account only metastable hadronic degrees of freedom in Eq. (2.4) – the baryon octet  $B = \{N, \Lambda, \Sigma, \Xi\}$  and the pseudoscalar nonet  $M = \{\pi, K, \eta, \eta'\}$  – we obtain a meaningful result, namely,  $\varepsilon_{vac(top)} + \varepsilon_{vac(h)} = 0$  for a reasonable cut-off parameter value  $\mu \simeq 1.2$  GeV. It turns out that the topological and collective quantum-wave fluctuations contribute to the QCD vacuum energy density with *opposite* signs. Therefore, a particular matching of numerical values of the non-perturbative QCD parameters (e.g. light hadron masses and scale parameters) via yet unknown dynamical mechanism could, in principle, provide zeroth net value of the non-perturbative QCD vacuum energy density without incorporating any extra physics at different space-time scales.

Note, the hypothesis about the exact cancellation of quantum-topological and quantum-wave contributions to the vacuum energy as an internal feature of the theory of non-perturbative QCD vacuum does not mean that the sum of their quantum fluctuations is

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<sup>2</sup> The numerical values for boundaries in Eq. (2.3) can be somewhat model-dependent which, in practice, would not affect any of our conclusions here.

identically equal to zero. Indeed, let us consider the complete unordered two-point function

$$\langle 0 | \frac{\alpha_s}{\pi} F_{ik}^a(x) F_a^{ik}(x') | 0 \rangle \equiv \langle 0 | : \frac{\alpha_s}{\pi} F_{ik}^a(x) F_a^{ik}(x') : | 0 \rangle + \langle 0 | \frac{\alpha_s}{\pi} F_{ik}^a(x) F_a^{ik}(x') | 0 \rangle_{(h)}. \quad (2.5)$$

The first term in Eq. (2.5) is expressed via the experimentally measured value (introduced for the first time in Ref. [24]) and a correlation function  $D(x)$  which can be constrained e.g. in lattice QCD or effective field theory methods, i.e. [38]

$$\begin{aligned} \langle 0 | : \frac{\alpha_s}{\pi} F_{ik}^a(x) F_a^{ik}(x') : | 0 \rangle &= \langle 0 | : \frac{\alpha_s}{\pi} F_{ik}^a(0) F_a^{ik}(0) : | 0 \rangle D_{top}(x - x'), \quad D_{top}(0) = 1, \\ \langle 0 | : \frac{\alpha_s}{\pi} F_{ik}^a(0) F_a^0(0) : | 0 \rangle &= (360 \pm 20 \text{ MeV})^4. \end{aligned} \quad (2.6)$$

For a detailed overview of the methods of calculation of higher power corrections to non-local condensates in QCD, see e.g. Ref. [39]. The second term in Eq. (2.5) representing the quantum-wave component of the two-point function has an analogical representation. Now, the hypothesis about the exact cancellation of topological and quantum-wave contributions written in terms of normally ordered correlation functions

$$\langle 0 | : \frac{\alpha_s}{\pi} F_{ik}^a(0) F_a^{ik}(0) : | 0 \rangle = -\langle 0 | \frac{\alpha_s}{\pi} F_{ik}^a(0) F_a^{ik}(0) | 0 \rangle_{(h)}$$

leads to the following expression for the sum of corresponding fluctuations:

$$\begin{aligned} \langle 0 | \frac{\alpha_s}{\pi} F_{ik}^a(x) F_a^{ik}(x') | 0 \rangle &= \langle 0 | : \frac{\alpha_s}{\pi} F_{ik}^a(0) F_a^{ik}(0) : | 0 \rangle D(x - x'), \\ D(x - x') &= D_{top}(x - x') - D_h(x - x'). \quad D(0) = 0. \end{aligned} \quad (2.7)$$

As was mentioned earlier, the fluctuations of both types occur at the same space-time scale. However, identical vanishing of the function (2.7) can not be assumed here, i.e. a completely different space-time dynamics of the quantum-topological and quantum-wave fluctuations, of course, does not imply that  $D(x - x') \equiv 0$  identically.

Thus, we have illustrated another possibility to eliminate the non-perturbative QCD contribution to the ground state energy of the Universe based on the hypothesis about a specific fine-tuning of the QCD vacuum parameters. Both macroscopic previously discussed in Ref. [29] and microscopic mechanisms considered above in this Section have a non-perturbative nature imprinted in essentially unknown quantum dynamics of the QCD vacuum, and the real situation can be, in principle, a superposition of both cancellation mechanisms (not excluding other possibilities, of course).

Once the QCD vacuum energy contribution has been eliminated to the leading order in metric perturbations, the observable small  $\Lambda$ -term density can be further generated by quantum-gravity corrections according to the SZ scenario [19, 20], i.e. by quantum metric fluctuations dynamically induced by the non-zeroth non-perturbative gluon field fluctuations described by Eq. (2.7). We will further demonstrate this fact by using the quasiclassical approximation methods in the conventional General Relativity and quantum gravity frameworks.

### III. QCD VACUUM ENERGY: FIRST ORDER IN METRIC PERTURBATIONS

#### A. Equations of motion for metric fluctuations and macroscopic geometry

In this Section, we briefly overview the quasiclassical (semiquantum) gravity framework in four dimensions, where the Zeldovich-Sakharov scenario [19, 20] can be mathematically

realized in the simplest and well-grounded way.

The semiclassical approach to quantum gravity deals with quantum fields defined on a classical background [40]. Typically, one starts from the action of the gravitational and external physical fields written in terms of the corresponding quantum field operators as follows

$$S = \int L d^4x, \quad L = -\frac{1}{2\kappa} \sqrt{-\hat{g}} \hat{g}^{ik} \hat{R}_{ik} + L(\hat{g}^{ik}, \chi_A), \quad (3.1)$$

where  $\hat{g}^{ik}$  and  $\hat{R}_{ik}$  are the metric and curvature operators, respectively;  $L(\hat{g}^{ik}, \chi_A)$  is the Lagrangian density of external physical (e.g. gauge) fields  $\chi_A$  with spins  $J < 2$  interacting with each other and with gravity; index  $A$  numerates degrees of freedom of these external fields.

In the framework of quasiclassical gravity theory it is conventionally assumed [40] that the metric operator  $\hat{g}^{ik}$  contains  $c$ -number part  $g^{ik}$  – the macroscopic space-time metric, and operator part  $\Phi_i^k$  – the quantum graviton field. In what follows, we work in the Heisenberg representation. This means that one introduces an extra postulate about the existence of the Heisenberg state vector  $|0\rangle$ , which contains information about initial states of all incident quantum fields. Then, the graviton field operator  $\Phi_i^k$ , by definition, satisfies the following condition on its average over the Heisenberg states:

$$\langle 0 | \Phi_i^k | 0 \rangle = 0. \quad (3.2)$$

As usual, derivation of the quasiclassical gravity theory equations of motion includes variation and averaging operations. In order to match the classical evolution of the macroscopic metric with quantum dynamics of gravitons it is necessary to require that independent variations of the action (3.1) over  $g^{ik}$  (taken at  $\Phi_i^k = \text{const}$ ) and  $\Phi_i^k$  (taken at  $g^{ik} = \text{const}$ ) must lead to the same operator equations, namely,

$$\delta \int L d^4x = -\frac{1}{2} \int d^4x \left( \sqrt{-g} \delta g^{ik} \hat{G}_{ik} \right)_{\Phi_i^k = \text{const}} = -\frac{1}{2} \int d^4x \left( \sqrt{-g} \delta \Phi^{ik} \hat{G}_{ik} \right)_{g^{ik} = \text{const}} = 0, \quad (3.3)$$

giving rise to

$$\begin{aligned} \hat{G}_i^k &= \frac{1}{2} (\delta_l^k \delta_i^m + g^{km} g_{il}) \left( \frac{\hat{g}}{g} \right)^{1/2} \hat{E}_m^l = 0, \\ \hat{E}_m^l &= \frac{1}{\kappa} \left( \hat{g}^{lp} \hat{R}_{pm} - \frac{1}{2} \delta_m^l \hat{g}^{pq} \hat{R}_{pq} \right) - \hat{g}^{lp} \hat{T}_{pm}(\hat{g}^{ik}, \chi_A), \end{aligned} \quad (3.4)$$

where  $T_{pm}(\hat{g}^{ik}, \chi_A)$  is the operator analog of the classical energy-momentum tensor corresponding to the Lagrangian  $L(\hat{g}^{ik}, \chi_A)$ . By averaging the operator equations (3.4) over the Heisenberg states, one obtains the equations of motion for the macroscopic (background) metric  $g^{ik}$ :

$$\langle 0 | \hat{G}_i^k | 0 \rangle = 0. \quad (3.5)$$

Subtracting  $c$ -number part (3.5) from operator (3.4), one obtains the equations of motion for the graviton fields  $\Phi_i^k$ :

$$\hat{G}_i^k - \langle 0 | \hat{G}_i^k | 0 \rangle = 0. \quad (3.6)$$

In addition to Eqs. (3.5) and (3.6), one also gets the operator equations of motion for external fields with  $J < 2$  by means of variations of the action (3.1) over  $\chi_A$ .



An explicit form of the functional  $\hat{G}_i^k$  can be obtained by a variation of the action over the macroscopic metric  $g^{ik}$  without implying an explicit form for the quantum  $\hat{g}^{ik}$  ( $g^{lm}$ ,  $\Phi_n^s$ ) operator. The condition (3.3) providing consistency of equations (3.5) and (3.6) fixes the exponential parameterisation for the metric operator as follows [41]

$$\sqrt{-\hat{g}}\hat{g}^{ik} = \sqrt{-g}g^{il}(\exp \psi)_l^k = \sqrt{-g}g^{il} \left( \delta_l^k + \psi_l^k + \frac{1}{2}\psi_l^m\psi_m^k + \dots \right), \quad (3.7)$$

where a shorthand notation for the graviton field has been introduced

$$\psi_i^k = \Phi_i^k - \frac{1}{2}\delta_i^k\Phi.$$

Applying the exponential parameterization (3.7), operator  $\hat{G}_i^k$  can then be transformed to the following compact expression:

$$\begin{aligned} \hat{G}_i^k &= \frac{1}{2\kappa} \left( \psi_{i;l}^{k;l} - \psi_{i;l}^{l;k} - \psi_{l;i}^{k;l} + \delta_i^k \psi_{l;m}^{m;l} + \psi_i^l R_l^k + \psi_l^k R_i^l - \delta_i^k \psi_l^m R_m^l \right) \\ &+ \frac{1}{\kappa} \left( R_i^k - \frac{1}{2}\delta_i^k R \right) - \hat{T}_i^k, \end{aligned} \quad (3.8)$$

where  $R_i^k$  is the Ricci tensor of macroscopic space-time with the background metric  $g_{ik}$  in which all the covariant derivatives and lowering/raising index operations are defined; and

$$\hat{T}_i^k = \hat{T}_{i(G)}^k + \frac{1}{2} \left( \delta_l^k \delta_i^m + g^{km} g_{il} \right) \left( \frac{\hat{g}}{g} \right)^{1/2} \hat{g}^{lp} \hat{T}_{pm} (\hat{g}^{ik}, \chi_A) \quad (3.9)$$

is the total energy-momentum tensor operator incorporating the graviton field contribution

$$\begin{aligned} \hat{T}_{i(G)}^k &= \frac{1}{4\kappa} \left( \psi_{m;i}^l \psi_l^{m;k} - \frac{1}{2} \psi_{;i} \psi^{;k} - \psi_{i;m}^l \psi_l^{m;k} - \psi_l^{k;m} \psi_{m;i}^l \right) \\ &- \frac{1}{8\kappa} \delta_i^k \left( \psi_{m;n}^l \psi_l^{m;n} - \frac{1}{2} \psi_{;n} \psi^{;n} - 2 \psi_{n;m}^l \psi_l^{m;n} \right) \\ &- \frac{1}{4\kappa} \left( 2 \psi_n^l \psi_i^{k;n} - \psi_n^k \psi_i^{l;n} - \psi_i^n \psi_{;n}^{kl} + \psi_i^{n;k} \psi_n^l + \psi_{n;i}^k \psi^{nl} + \delta_i^k (\psi_m^n \psi_n^l)^{;m} \right)_{;l} \\ &- \frac{1}{4\kappa} \left( \psi_i^m \psi_n^l R_l^k + \psi_n^k \psi_l^n R_i^l - \delta_i^k \psi_l^n \psi_n^m R_m^l \right) + O(\psi^3). \end{aligned} \quad (3.10)$$

Averaging the operator  $\hat{G}_i^k$  given by Eq. (3.8) with an extra defining condition on the quantum graviton field  $\Phi_i^k$  (3.2), we see that the equations of motion for the macroscopic (background) metric  $g_{ik}$  (3.5) are transformed into usual Einstein equations as expected

$$\frac{1}{\kappa} \left( R_i^k - \frac{1}{2} \delta_i^k R \right) = \langle 0 | \hat{T}_i^k | 0 \rangle, \quad (3.11)$$

where a macroscopic average  $\langle 0 | \hat{T}_i^k | 0 \rangle$  in the right hand side contains a contribution from the energy-momentum tensor of the quantum graviton field  $\hat{T}_{i(G)}^k$  according to Eq. (3.9). Applying the same procedure to Eq. (3.6) using expression (3.8), we finally obtain the equations of motion for graviton fields in explicit form:

$$\psi_{i;l}^{k;l} - \psi_{i;l}^{l;k} - \psi_{l;i}^{k;l} + \delta_i^k \psi_{l;m}^{m;l} + \psi_i^l R_l^k + \psi_l^k R_i^l - \delta_i^k R_l^m \psi_m^l = 2\kappa \left( \hat{T}_i^k - \langle 0 | \hat{T}_i^k | 0 \rangle \right). \quad (3.12)$$

A more rigorous derivation of the equations of motion (3.11) and (3.12) is based on the canonical quantum gravity in the path integral (Faddeev-Popov) formulation [42]. In order to turn from the complete quantum gravity to its quasiclassical (semiquantum) limit, one starts from factorization of the path integral measure provided by the exponential parametrization of the metric operator (3.7). A subsequent calculation of the factorized path integral, which is exact over the high-frequency (quantum) fields  $\Phi_i^k$  and saddle-point approximated over the slow-changing (classical) fields  $g_{ik}$ , is equivalent to solving the equations of motion in the operator formulation (3.11) and (3.12). This demonstrates theoretical consistency of the operator approach described above.

## B. Operator gluodynamics with vacuum anomaly

The non-perturbative fluctuations of gluon and quark fields naturally gravitate and should be included into equations of motion in the framework of operator field dynamics (3.11) and (3.12). The non-perturbative dynamics of gluon and quark fields is not sufficiently well developed in the literature, so we build up our analysis based upon the functional relations between metric fluctuations and vacuum fluctuations of quark and gluon fields only. These relations can be obtained from Eq. (3.12) and imply the existence of an adequate field-theoretical model for the QCD vacuum energy-momentum tensor incorporating conformal anomalies. The recipe for getting such a tensor is demonstrated e.g. in Ref. [35]: within the variational procedure for getting the energy-momentum tensor and equations of motion, the QCD coupling  $g_s$  can be viewed as an operator depending on operators of quantum fields according to the Renormalisation Group (RG) equations. In the framework of this formalism, one introduces the gluon field (vector-potential) operator  $\mathcal{A}_i^a$ , as a variational variable, related to the gluon field operator in the standard normalisation as follows  $\mathcal{A}_i^a = g_s A_i^a$ . The stress tensor operator is then defined as  $\mathcal{F}_{ik}^a = \partial_i \mathcal{A}_k^a - \partial_k \mathcal{A}_i^a + f^{abc} \mathcal{A}_i^b \mathcal{A}_k^c$ . In the case of pure gluodynamics, the QCD coupling operator  $g_s^2 = g_s^2(J)$  depends upon the gauge operator of least dimension  $J \equiv \mathcal{F}_{ik}^a \mathcal{F}_a^{ik}$  by means of the operator RG evolution equation

$$2J \frac{dg_s^2(J)}{dJ} = g_s^2(J) \beta[g_s^2(J)], \quad (3.13)$$

where  $\beta[g_s^2(J)]$  is the QCD  $\beta$ -function calculable in the standard quantum field theory framework. A solution of Eq. (3.13) is substituted into the effective operator Lagrangian

$$L_{\text{eff}} = -\frac{1}{4g_s^2(J)} \mathcal{F}_{ik}^a \mathcal{F}_a^{ik}, \quad (3.14)$$

whose variation w.r.t  $\mathcal{A}_i^a$  leads to the operator energy-momentum tensor of the gluon field

$$\hat{T}_{i(g)}^k = \frac{1}{g_s^2(J)} \left( -\mathcal{F}_{il}^a \mathcal{F}_a^{kl} + \frac{1}{4} \delta_i^k \mathcal{F}_{ml}^a \mathcal{F}_a^{ml} + \frac{\beta[g_s^2(J)]}{2} \mathcal{F}_{il}^a \mathcal{F}_a^{kl} \right) \quad (3.15)$$

and operator equation of motion of pure gluodynamics

$$D_k^{ab} \left\{ g_s^{-2}(J) \left( 1 - \frac{\beta[g_s^2(J)]}{2} \right) \mathcal{F}_b^{ik} \right\} = 0, \quad (3.16)$$

$$D_k^{ab} = \delta^{ab} \partial_k - f^{abc} \mathcal{A}_k^c.$$

After a straightforward covariant generalization, the operator gluodynamics defined by Eqs. (3.15) and (3.16) can be incorporated into the quasiclassical (semiquantum) gravity in the Heisenberg operator formulation given by Eqs. (3.11) and (3.12). In the framework of this formulation it is crucial that the energy-momentum tensor (3.15) is conservative under the operator equations of motion (3.16).

In the one-loop approximation we have

$$\beta[g_s^2(J)] = -\frac{bg_s^2(J)}{16\pi^2}, \quad \frac{g_s^2(J)}{4\pi} \equiv \alpha_s(J) = \frac{8\pi}{b \ln(J/\lambda^4)}, \quad (3.17)$$

where  $\lambda$  is the QCD scale parameter discussed below;  $b = b(0) = 11$  is the one-loop  $\beta$ -function coefficient of the pure gluodynamics (without quark fields).

In order to construct a realistic operator energy-momentum tensor in QCD one has to incorporate quark fields. Formally, inclusion of the quark fields lead to extra terms in the energy-momentum tensor and to new operator equations of motion. However, this procedure can be simplified within the adopted phenomenological approach. It is clear from the beginning that an account for quark fields changes the numerical values of the  $\beta$ -function coefficients. Also, non-perturbative quark-gluon fluctuations happen at a characteristic scale of four-momentum transfers smaller than the double charm quark mass, so one has to fix  $b = b(3) = 9$  in Eq. (3.17) and, correspondingly, in Eqs. (3.15) and (3.16). Further, an induced character of quark fluctuations in the quark sea (effectively arising mostly due to gluon splittings in  $q\bar{q}$  pairs at small momentum transfers) provides that QCD vacuum observables can be approximately expressed through the square of averaged gluon field fluctuations. In particular, for quantum-topological quark-gluon fluctuations it is well known that [32]

$$\langle 0 | : \bar{s}s : | 0 \rangle \simeq \langle 0 | : \bar{u}u : | 0 \rangle = \langle 0 | : \bar{d}d : | 0 \rangle = -\langle 0 | : \frac{\alpha_s}{\pi} F_{ik}^a F_a^{ik} : | 0 \rangle L_g = -(225 \pm 25 \text{ MeV})^3, \quad (3.18)$$

where  $L_g \simeq (1500 \pm 300 \text{ MeV})^{-1}$  is the correlation length of fluctuations which is normally calculated through the experimental data on quark and gluon condensates (and supported by the lattice QCD calculations). Its value is close to the minimal scale of fluctuations given in Eq. (2.3) at which their level is maximal. Within phenomenologically reasonable assumptions discussed above in Sect. II, the non-perturbative quantum-wave (hadron) fluctuations occur at the same space-time scales, thus they should satisfy to a functional relation analogical to Eq. (3.18). Under these assumptions the operator relation between quark and gluon fluctuations can be established by taking the trace of the averaged quark energy-momentum tensor and then applying its conservation condition and relations (2.2) and (3.18) valid for both quantum-topological and quantum-wave contributions. This procedure provides us with the effective quark contribution to the QCD energy momentum tensor in the following form:

$$\hat{T}_{i(q),\text{eff}}^k = \frac{8L_g}{b(3)}(m_u + m_d + m_s)\hat{T}_{i(g)}^k. \quad (3.19)$$

Finally, adding Eqs. (3.15) and (3.19) written in the one-loop approximation gives phenomenologically motivated complete QCD energy-momentum tensor in operator form

$$\hat{T}_{i(\text{QCD})}^k \simeq \frac{b_{\text{eff}}}{32\pi^2} \left( -\mathcal{F}_{il}^a \mathcal{F}_a^{kl} + \frac{1}{4} \delta_i^k \mathcal{F}_{ml}^a \mathcal{F}_a^{ml} \right) \ln \frac{eJ}{\lambda^4} - \delta_i^k \frac{b_{\text{eff}}}{128\pi^2} \mathcal{F}_{ml}^a \mathcal{F}_a^{ml}, \quad (3.20)$$

$$b_{\text{eff}} = b(3) + 8L_g(m_u + m_d + m_s) \simeq 9.6,$$

and the operator equation of motion

$$D_k^{ab} \left( \mathcal{F}_b^{ik} \ln \frac{eJ}{\lambda^4} \right) = 0. \quad (3.21)$$

Of course, the resulting model expressions (3.20), (3.21) are approximate and restricted by quantitative phenomenological estimates (2.2) and (3.18) which characterize the non-perturbative quark-gluon fluctuations.

Further simplifications are possible and make use of series expansion of the logarithmic operator function in small fluctuations as follows

$$\ln \frac{eJ}{\lambda^4} = \ln \frac{e\langle 0|J|0\rangle}{\lambda^4} + \frac{J - \langle 0|J|0\rangle}{\langle 0|J|0\rangle} + \dots$$

Note, the logarithmic function in the energy-momentum tensor (3.20) comes as a multiplier to an expression which has zeroth vacuum expectation value. Thereby, an account for the leading correction to the logarithm means that in the energy-momentum tensor, together with leading terms linear in  $\delta J = J - \langle 0|J|0\rangle$ , one also takes into account the higher-order terms in  $\delta J$ . Since the color factor suppresses the mean square fluctuation of the logarithm by a factor of  $1/24$ , then replacement of the operator function under the logarithm by its averaged value can be considered as a good approximation. Under such a replacement,  $g_s^2(eJ)$  operator transforms into the usual QCD coupling at a characteristic scale of non-perturbative QCD fluctuations in the region of four-momentum transfers squared less than  $L_g^{-2} \simeq (1500 \text{ MeV})^2$ , i.e.

$$\ln \frac{e\langle 0|J|0\rangle}{\lambda^4} \simeq 4 \ln \frac{L_g^{-1}}{\Lambda_{\text{QCD}}},$$

where  $\Lambda_{\text{QCD}} \simeq 160 \text{ MeV}$  is the QCD scale parameter.

All subsequent calculations are controlled by the fact that the approximate expressions derived above must satisfy the energy-momentum tensor conservation. The replacement of the operator function under the logarithm by its averaged value transforms operator equations (3.21) into the standard Yang-Mills equations. To this approximation, the energy-momentum (3.20) should also transform (up to a constant multiplicative term and a constant additive term) into the standard Yang-Mills energy-momentum tensor. This result is achieved by a replacement of the conformal anomaly operator in the energy-momentum tensor (3.20) by its averaged value. Then, turning back to original symbols  $A_i^a = \mathcal{A}_i^a/g_s$ ,  $F_{ik}^a = \mathcal{F}_{ik}^a/g_s$  under the approximations adopted above, one writes finally

$$\hat{T}_{i(\text{QCD})}^k = \frac{b_{\text{eff}}\alpha_s}{2\pi} \left( -F_{il}^a F_a^{kl} + \frac{1}{4} \delta_i^k F_{ml}^a F_a^{ml} \right) \ln \frac{L_g^{-1}}{\Lambda_{\text{QCD}}} - \delta_i^k \frac{b_{\text{eff}}}{32} \langle 0 | \frac{\alpha_s}{\pi} F_{ml}^a F_a^{ml} | 0 \rangle. \quad (3.22)$$

$$D_k^{ab} F_b^{ik} = 0, \quad D_k^{ab} = \delta^{ab} \partial_k - g_s f^{abc} A_k^c. \quad (3.23)$$

As a matter of fact, we have not done anything radically new here – similar equations are often used in the Euclidean QCD framework where the vacuum energy-momentum tensor is estimated at instanton solutions of the classical Yang-Mills equations [32]. A proper covariant generalization of this framework will be applied for the  $\Lambda$ -term density calculation in the next Section.

#### IV. $\Lambda$ -TERM CALCULATION

Now, we have prepared everything what is needed for estimation of the dynamically induced  $\Lambda$ -term. In the right hand side of the macroscopic Einstein equations (3.11) the terms which correspond to the graviton-mediated interactions of the non-perturbative quark-gluon fluctuations are of the order of  $O(G)$  and  $O(\alpha_s G)$  with  $G$  being the gravitational constant. Of course, in order to estimate the leading order effect, it makes sense to take into account only the first-order (linear) non-vanishing terms in gravitational constant  $G$ . Furthermore, due to an obvious smallness of the typical QCD space-time scales compared to the cosmological scales, the induced quantum fluctuations of metric should be considered at the Minkowski background. At last, it is sufficient to consider only the trace of the macroscopic Einstein equations  $R + 4\kappa\varepsilon_\Lambda = 0$  giving rise to the QCD-induced  $\Lambda$ -term density

$$\varepsilon_\Lambda = -\frac{b_{\text{eff}}}{32}\langle 0|\frac{\alpha_s}{\pi}\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{il}\hat{g}^{km}\hat{F}_{ik}^a\hat{F}_{lm}^a|0\rangle + \frac{1}{4}\langle 0|\hat{T}_{(G)}|0\rangle. \quad (4.1)$$

Let us start with expansion of the gluon stress tensor in series over the metric fluctuations (gravitons). From Eq. (3.23) written in Riemann space

$$\left(\delta^{ab}\frac{\partial}{\partial x^k} - g_s f^{abc}\hat{A}_k^c\right)\sqrt{-\hat{g}}\hat{g}^{il}\hat{g}^{km}\hat{F}_{lm}^b = 0,$$

it follows immediately

$$\hat{F}_{ik}^a = F_{ik}^a + \frac{1}{2}\psi F_{ik}^a - \psi_i^l F_{lk}^a - \psi_k^l F_{il}^a + O(\alpha_s G), \quad (4.2)$$

where  $F_{ik}^a$  is the usual stress tensor at the macroscopic background which does not account for interactions between gluon field and metric fluctuations (gravitons). The expansion (4.2) can be used only for extraction of the leading-order effect in the first term of Eq. (4.1), which initially is of the order of  $O(\alpha_s)$ . Thereby, higher terms  $O(\alpha_s G)$  in the expansion (4.2) generate corrections to the  $\Lambda$ -term density (4.1) of the order of  $O(\alpha_s^2 G)$ , which go beyond the one-loop approximation adopted here and therefore are omitted.

The second term of Eq. (4.1) is formed by induced fluctuations of the metric (i.e. by a solution of equation (3.12)), which are unambiguously related with the gluon field fluctuations. To the leading order in gravitational constant  $G$ , in the left hand side of Eq. (3.12) we keep only proper fluctuations of the quadratic form  $F_{il}^a F_a^{kl}$  without taking into account gravity. Then, using Eq. (3.22) in the right hand side of Eq. (3.12), we obtain an important relation between the graviton field  $\psi_{ik}$  and the gluon field strength  $F_{ik}^a$  to the respective order:

$$\psi_{i,l}^{k,l} - \psi_{i,l}^{l,k} - \psi_{l,i}^{k,l} + \delta_i^k \psi_{l,m}^{m,l} = \frac{\kappa\alpha_s b_{\text{eff}}}{\pi} \left( -F_{il}^a F_a^{kl} + \frac{1}{4}\delta_i^k F_{ml}^a F_a^{ml} \right) \ln \frac{L_g^{-1}}{\Lambda_{\text{QCD}}}. \quad (4.3)$$

At the next step one substitutes the expansion (4.2) into the first term of Eq. (4.1). The second term in Eq. (4.1) can be calculated using the trace of the averaged energy-momentum tensor of gravitons (3.10) (the averaging removes total derivatives due to symmetry of the Minkowski space-time background). After the averaging, the zeroth-order term in metric fluctuations (given by the unperturbed trace of the quark-gluon vacuum energy-momentum tensor) disappears due to the hypothesis about exact cancellation of quantum-topological

and quantum-wave contributions discussed in Sect. II. Implying Eq. (4.3), the resulting non-vanishing effect in the  $\Lambda$ -term density turns out to be quadratic in the graviton field  $\psi_{ik}$ :

$$\begin{aligned} \varepsilon_\Lambda = & -\frac{b_{\text{eff}}}{64} \langle 0 | \left( \frac{\alpha_s}{\pi} F_{ml}^a F_a^{ml} \psi - 4 \frac{\alpha_s}{\pi} F_{nm}^a F_a^{lm} \psi_l^n \right) | 0 \rangle \\ & - \frac{1}{16\kappa} \langle 0 | \left( \psi_{m,n}^l \psi_l^{m,n} - \frac{1}{2} \psi_{,n} \psi^{,n} - 2 \psi_{n,m}^l \psi_l^{m,n} \right) | 0 \rangle. \end{aligned} \quad (4.4)$$

The second term in Eq. (4.4) can be identically transformed and simplified as follows: by transferring derivatives to one of the multipliers in each term one arrives at the differential form which can then be expressed through the left hand side of Eq. (4.3). Then applying the latter and combining all the terms together, the resulting  $\Lambda$ -term density takes a remarkably simple form convenient for phenomenological analysis:

$$\varepsilon_\Lambda = -\frac{b_{\text{eff}}}{16} \ln \frac{L_g^{-1}}{e\Lambda_{\text{QCD}}} \langle 0 | \frac{\alpha_s}{\pi} F_{il}^a F_a^{kl} \left( \psi_k^i - \frac{1}{4} \delta_k^i \psi \right) | 0 \rangle. \quad (4.5)$$

The simplest way to obtain the physically meaningful result is to fix the Fock gauge  $\psi_{i;k}^k = 0$ . In this case, according to Eq. (4.3),  $\psi = 0$ , so the final solution for the graviton field reads

$$\psi_i^k(x) = \kappa b_{\text{eff}} \ln \frac{L_g^{-1}}{\Lambda_{\text{QCD}}} \int d^4x' \mathcal{G}(x-x') \cdot \left( \frac{\alpha_s}{\pi} F_{il}^a(x') F_a^{kl}(x') - \delta_i^k \frac{\alpha_s}{4\pi} F_{ml}^a(x') F_a^{ml}(x') \right). \quad (4.6)$$

where  $\mathcal{G}(x-x')$  is the Green function satisfying the Green equation  $\mathcal{G}_{,l}^{:l} = -\delta(x-x')$ . After substitution of Eq. (4.6) into Eq. (4.5), the respective averages should be calculated according to the following rules:

$$\begin{aligned} \langle 0 | \frac{\alpha_s}{\pi} F_{il}^a(x) F_a^{kl}(x) \cdot \left( \frac{\alpha_s}{\pi} F_{km}^b(x') F_b^{im}(x') - \langle 0 | \frac{\alpha_s}{\pi} F_{km}^b(x') F_b^{im}(x') | 0 \rangle \right) | 0 \rangle = \\ \langle 0 | \frac{\alpha_s}{\pi} F_{il}^a(x) F_{km}^b(x') | 0 \rangle \langle 0 | \frac{\alpha_s}{\pi} F_a^{kl}(x) F_b^{im}(x') | 0 \rangle + \\ \langle 0 | \frac{\alpha_s}{\pi} F_{il}^a(x) F_b^{im}(x') | 0 \rangle \langle 0 | \frac{\alpha_s}{\pi} F_a^{kl}(x) F_{km}^b(x') | 0 \rangle, \end{aligned} \quad (4.7)$$

$$\langle 0 | \frac{\alpha_s}{\pi} F_{il}^a(x) F_{km}^b(x') | 0 \rangle = \frac{\delta^{ab}}{96} (g_{ik} g_{lm} - g_{im} g_{kl}) \langle 0 | : \frac{\alpha_s}{\pi} F_{nj}^c(0) F_c^{nj}(0) : | 0 \rangle D(x-x'). \quad (4.8)$$

The first rule, Eq. (4.7), demonstrates the exchange character of gravitational interactions of the gluon fluctuations, the first-order one in fluctuations (the zeroth-order single-point averages are explicitly subtracted). Also, Eq. (4.8), besides the gauge/Lorentz symmetry properties, incorporates the effect of compensation of the quantum-topological and quantum-wave contributions – the two-point function  $D(x-x')$  is defined as a difference between the corresponding correlation functions as reflected in Eq. (2.7). Substitution of these formulas into Eq. (4.5) leads to our final result for the QCD-induced dynamical  $\Lambda$ -term energy density:

$$\begin{aligned} \varepsilon_\Lambda = & -\pi G \langle 0 | : \frac{\alpha_s}{\pi} F_{ik}^a F_a^{ik} : | 0 \rangle^2 \times \left( \frac{b_{\text{eff}}}{8} \right)^2 \ln \frac{L_g^{-1}}{e\Lambda_{\text{QCD}}} \ln \frac{L_g^{-1}}{\Lambda_{\text{QCD}}} \int d^4y \mathcal{G}(y) D^2(y) = \\ & = (1 \pm 0.5) \times 10^{-29} \Delta \text{ MeV}^4. \end{aligned} \quad (4.9)$$

Here, we introduced the dimensionless parameter

$$\Delta = -\frac{1}{L_g^2} \int d^4y \mathcal{G}(y) D^2(y), \quad (4.10)$$

which is defined in the Euclidean 4-space such that  $\Delta > 0$ . Its numerical value has to be established by a dynamics in a complete non-perturbative QCD vacuum theory, or estimated in the static approximation e.g. in lattice QCD or in effective field theory approaches.

In general, the expression (4.9) provides a good estimate for the dynamical contribution of the graviton-exchange interactions in the hadronic vacuum to the vacuum energy density of the Universe, it is unavoidable and should be taken into consideration in any theoretical or phenomenological analysis of the Dark Energy.

## V. DISCUSSION AND CONCLUSIONS

The scenario for the  $\Lambda$ -term generation suggested above is realistic under two necessary and sufficient conditions:

- the *exact compensation* of the non-perturbative quantum-topological and quantum-wave contributions to the QCD vacuum energy density in the *zeroth order in quantum gravity* effects;
- the *almost exact compensation* of the graviton-exchange interactions of topological and wave fluctuations with a *small residual effect* summarized in Eq. (4.9) which can be identified with the observable  $\Lambda$ -term.

The physical and mathematical premises for the first condition are contained in physically reasonable estimates for the energy density possibly coming from the finite-time instanton solutions of the cosmological Yang-Mills fields as discussed in Ref. [29] and/or from vacuum hadronic wave modes given by Eq. (2.4). Both of these estimates show that the respective contributions compensating a large negative instanton energy (2.2), may have a similar nature being components of the same *non-perturbative QCD vacuum*. An interesting analogy of the co-existence of vacuum contributions of essentially different types, but of the same origin, takes place e.g. in solid state physics: in a crystallization process there are both negative (binding energy of atoms in the crystal lattice) and positive (zero-point collective fluctuations of the lattice itself) contributions to the “vacuum” energy of the medium. Of course, such an analogy is not exact and there is no exact compensation of these contributions in this case.

The problem of exact compensation of vacuum contributions at the QCD energy scale seemingly has a dynamical nature. No doubts, the QCD vacuum state in the modern Universe has been created during its cosmological evolution: the quantum-topological vacuum structures (e.g. instantons) have been created in real time around the quark-hadron phase transition in the Universe evolution. The quantum state of vacuum collective (wave) fluctuations of such structures should, therefore, be matched to the quantum state of these structures themselves. At the same time, the unprecedented accuracy in cancellation between different vacuum contributions to many relevant digits (or their major fine tuning), if it takes place in Nature, forces us to consider such a matching of the vacuum topological structures and their collective fluctuations as a new physical phenomenon. Most probably,

an adequate description of this phenomenon is the subject of a new dynamical theory of the non-perturbative QCD vacuum which has not been yet created.

The second condition above is tightly connected to the first one. Indeed, as is seen from Eqs. (2.5), (2.6) and (2.7), the representation of the two-point function  $D(x - x')$  as a difference between topological and hadron correlation functions appears as a consequence of our assumption about the exact compensation of the topological and hadron wave contributions to the vacuum energy density calculated without the graviton-mediated corrections. As we will see below, the smallness of the QCD parameter  $\Delta \sim 3 \cdot 10^{-6}$  introduced in Eq. (4.10) is required by phenomenological arguments – both by data on the  $\Lambda$ -term density itself and by QCD phenomenology. On the other hand, the parameter  $\Delta$  is a functional of the difference between the correlation functions defined at the same space-time scales (2.3). Thereby, there are physical and mathematical premises for the mutual cancellation of topological and wave contributions also in the effect graviton-mediated interactions, but such a cancellation is not exact. Thus, without a self-consistent theory of the non-perturbative QCD vacuum the actual value of the residual effect and, hence, the genuine value of  $\Lambda$ -term density can only be estimated by means of available experimental data and phenomenological arguments only. Based on purely phenomenological QCD arguments, let us try to move on towards a simple numerical estimation of the  $\Lambda$ -term density starting from Eq. (4.9).

Indeed, the small parameter  $\Delta \sim 3 \cdot 10^{-6}$  can, in principle, be expressed through the well-known QCD parameters. Substituting only topological  $D_{top}(x - x')$  or hadronic  $D_h(x - x')$  part of the complete correlation function  $D(x - x')$  into Eq. (4.10) one immediately obtains  $\Delta \sim 1$ . Meanwhile, it is known that naturally small QCD parameters are the light quark  $u, d, s$  masses, so let us assume that the value of (4.10) with the complete QCD correlation function  $D(x - x')$  is different from zero due to the *chiral symmetry breaking* effects. A small shift between the characteristic scales of topological and hadronic fluctuations induced by the chiral symmetry breaking can be given in terms of small current quark  $m_u, m_d, m_s$  masses

$$\begin{aligned} 1/L_{top} &\sim 1/L_h \sim 1/L_g, \\ |1/L_{top} - 1/L_h| &\sim m_u + m_d + m_s, \end{aligned}$$

leading to a good estimate

$$\Delta = k \cdot \frac{(m_u + m_d + m_s)^2 L_g^2}{(2\pi)^4} > 0, \quad (5.1)$$

where  $k \sim 1$  is the dimensionless factor, and the factor  $1/(2\pi)^4$  appears in the Fourier transform of the corresponding Green function. Then, the experimentally observable value of the  $\Lambda$ -term density (1.1) can be naturally obtained for  $k = 1$  and the light quark masses satisfying the approximate sum rule  $m_u + m_d + m_s \simeq 100$  MeV, which is in agreement with experimentally known values  $m_u = 1.5 - 5$  MeV,  $m_d = 3 - 9$  MeV,  $m_s = 60 - 170$  MeV. In this case, the phenomenological formula for the  $\Lambda$ -term density, obtained based upon Eqs. (4.9) and (5.1), contains all the basic QCD and chiral symmetry breaking parameters, i.e.

$$\varepsilon_\Lambda = \pi G \langle 0 | : \frac{\alpha_s}{\pi} F_{ik}^a F_a^{ik} : | 0 \rangle^2 \times \left( \frac{b_{\text{eff}}}{8} \right)^2 \frac{(m_u + m_d + m_s)^2 L_g^4}{(2\pi)^4} \ln \frac{L_g^{-1}}{e\Lambda_{\text{QCD}}} \ln \frac{L_g^{-1}}{\Lambda_{\text{QCD}}}. \quad (5.2)$$

In fact, Eq. (5.2) provides a naive estimate for the dynamical QCD-induced *positive* contribution to the  $\Lambda$ -term energy density  $\varepsilon_\Lambda > 0$  which is close to the observed value within



a factor of few. For a better estimate, one would certainly like to evaluate the  $\Delta$ -factor in Eq. (4.9) to a higher precision incorporating the chiral symmetry breaking effects e.g. in the lattice QCD framework or elsewhere.

To conclude, we notice that under the above two conditions the  $\Lambda$ -term generation at the QCD energy scale appears to be a natural phenomenon for supersymmetric (or superstring) scenarios of high-energy physics (but would be confronted, perhaps, by Technicolor models). Indeed, within these scenarios the vacuum subsystem of a quantum-topological nature exists only in the QCD sector. All other vacuum subsystems consist mainly of perturbative (weakly-coupled) vacuum fluctuations of fundamental fields and one may expect that e.g. a proper supersymmetry theory is capable of explaining the exact compensation of their contributions into the  $\Lambda$ -term density if  $\sum_{\text{bos}} M_{\text{bos}}^6 \equiv \sum_{\text{ferm}} M_{\text{ferm}}^6$  is satisfied. This can be seen e.g. from Eq. (1.1) where the respective energy scale (virtual particle mass of bosons  $M_{\text{bos}}$  and fermions  $M_{\text{ferm}}$ ) appears to the sixth power while bosons and fermions contribute to the vacuum energy with opposite signs. A detailed study of cancellations of this type is planned for a forthcoming study.

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